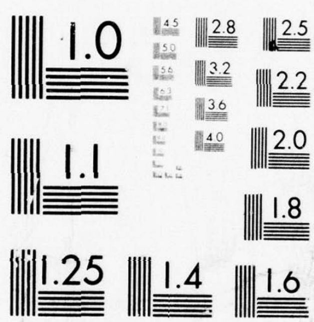
AD-A051 321 MASSACHUSETTS INST OF TECH LEXINGTON LINCOLN LAB F/G 9/3
A COMPARISON OF THE USE OF DETERMINISTIC AND RANDOM SIGNALS FOR--ETC(U)
DEC 77 D H PRUSLIN F19628-78-C-0002
UNCLASSIFIED TN-1977-49 ESD-TR-77-344 NL

| OF |

AD
A051321

END
DATE
FILMED
4-78
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

12

Technical Note 1977-49

D. H. Pruslin

A Comparison of the Use of Deterministic and Random Signals for Impulse Response Determination

27 December 1977

Prepared for the Department of the Army
under Electronic Systems Division Contract F19628-78-C-0002 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LEXINGTON, MASSACHUSETTS

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology. This program is sponsored by the Ballistic Missile Defense Program Office, Department of the Army; it is supported by the Ballistic Missile Defense Advanced Technology Center under Air Force Contract F19628-78-C-0002.

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

Raymond L. Loiselle

Raymond L. Loiselle, Lt. Col., USAF
Chief, ESD Lincoln Laboratory Project Office

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

A COMPARISON OF THE USE OF DETERMINISTIC AND RANDOM SIGNALS FOR IMPULSE RESPONSE DETERMINATION

D. H. PRUSLIN

Group 32

TECHNICAL NOTE 1977-49

27 DECEMBER 1977

LEXINGTON MASSACHUSETTS

PRECEDING PAGE BLANK - FILMED

Contents

I.	Introduction	1
	A. Origin of Problem	1
	B. Deterministic Signal Approach	2
	C. Random Signal Approach	5
	D. Relationship of Approaches	7
II.	Analysis	10
	A. Framework	10
	1. Performance Measure	10
	2. Model Considerations	13
	3. Operating Regime	14
	B. Random Signal Method	17
	1. Continuous Impulse Response	19
	a. Var_x Term	21
	b. Var_n Term	23
	2. Impulsive Impulse Response	26
	a. Var_x Term	27
	b. Var_n Term	30
	C. Deterministic Signal Method	31
	1. Continuous Impulse Response	34
	2. Impulsive Impulse Response	38
III.	Conclusions	39
	Appendix	41
	Acknowledgment	47

Preceding Page Blank - Filmed

I. Introduction

A. Origin of Problem

Suppose there come into your possession one or more "black boxes" and the information that these are linear. Why would you want to determine their impulse responses, and how would you go about it?

A fairly literal case would be that you had purchased the systems "impulsively" at a surplus shop, and discovered that the innards were potted so as to make direct inspection impossible. It would then be desirable to measure the impulse response or the transfer function (its Fourier transform) of each system to determine how it could be utilized.

In a ballistic missile defense (BMD) context the linear systems in question may model how projectiles of interest to the defense, re-entry vehicles (RV's), decoys, chaff particles, etc. respond in time to the voltage waveform transmitted by a radar. One of the defense functions is discrimination - the task of determining which targets must be acted against and which are there to confuse or saturate the defense. One of the methods for doing this is to utilize differences in the size or shape of the targets. Direct inspection is again impossible, so the received radar waveform is used. Sizing a target along the radar line of sight amounts to explicitly or implicitly determining and then further operating on the impulse response of a linear system. If the processing technique extracts an estimate of the target extent along the line of sight it is referred to as "length measurement." If the processing makes more implicit use of the impulse response it is a particular application of a technique called "pattern recognition." If sizing is also carried out in the cross-range direction using Doppler processing the technique is referred to as "imaging." We will restrict ourselves in what follows to the one-dimensional case.

B. Deterministic Signal Approach

Having decided that you want to determine an impulse response, how do you proceed? The basic procedure is to drive the system with a test signal, observe the response and process it, the processing depending on the test signal. The most obvious approach is to "pulse" the system. A deterministic pulse narrow with respect to the system response time (wideband with respect to the system bandwidth) is applied, and the resulting output signal is taken as an estimate of the impulse response, albeit somewhat smeared and distorted. Choosing an appropriate pulse bandwidth requires prior knowledge about the system or a trial and error procedure.

In principle, the effects of finite pulse bandwidth and detailed pulse-shape can be removed either in the time or frequency domains, yielding the "true" impulse response. In practice the degree to which this can be done is limited by interfering noise, and while the effect of interfering noise is the main subject of this report, it is assumed that this processing refinement is not involved.

In the frequency domain the unknown transfer function could be traced out by applying a sinusoid to the system and varying its frequency. The use of a pulse waveform can be thought of as a way of simultaneously applying frequency components to the system. This report will deal with the time domain version of the problem.

What if we allow the simple pulse to be less simple? That is, what if we employ a signal having the desired bandwidth but a time-bandwidth product greater than unity? The basic reasons for employing such a waveform in radar practice are: (i) the presence of interfering noise originating within the radar receiver and possibly also from electronic countermeasures (ECM), external jamming sources employed by the offense to screen targets from the radar (ii) that the achievable signal-to-noise ratio or radar sensitivity depends on how much energy can be packed into the radar signal (iii) and that radar transmitters operate under a peak power level constraint.

Thus a modulation such as linear frequency modulation (LFM), or binary phase shift coding is utilized to increase the signal duration and hence energy while maintaining the bandwidth needed for resolution. The receiver processing must now change from a simple observation of the returned pulse shape for two reasons.

Firstly, this pulse-shape no longer looks almost like the test impulse response, due to the longer signal duration. To correct this the signal bandwidth must be utilized to achieve its inherent time resolution. The conventional way of doing this is to convert the radar signal into a signal having a unity time-bandwidth product by matched filtering, i.e., by convolving the received signal with a time-reversed replica of the transmitted signal. It is well known that matched filtering generates the autocorrelation function of the signal in question, which in fact has a central "spike" whose time duration is the reciprocal of the signal bandwidth. The overall effect is that of having transmitted a "narrow pulse," the autocorrelation function of the signal, and taken the response of the unknown system to this waveform as an estimate of its impulse response.

Secondly, the receiver processing must take into account the interfering noise which dictated the use of a time extended modulated signal from an energy standpoint. Basically the receiver will reject noise outside the signal bandwidth, and secondarily it will employ an optimized passband shape to maximize the signal-to-noise ratio. In other words, it will employ the same matched filter that was required to achieve the desired resolution from the radar signal.

The above paragraphs have described how a conventional pulse compression radar would be used to measure an impulse response. In the analysis to be described in Section II, a scalar or baseband (rather than complex or bandpass) model is used for the time domain signals and systems. Thus in the pulsed case the details of a phase modulated pulse compression radar are not modeled. Rather the transmitted signal is thought of as a "narrow" pulse whose time-bandwidth product is approximately unity, and the receiver as employing a filter matched to that signal. This simplification in no way compromises the applicability of the results to a "real" radar system.

C. Random Signal Approach

Let us now shift to a consideration of random signals and their possible application to impulse response determination. As we have noted, the use of a pulse signal can be thought of as a way of simultaneously applying many frequency components to trace out the unknown transfer function. Can a random process be used to perform this function, and if so why would it be desirable to use one? The familiar stationary "whitish" Gaussian process so often employed in radar analysis can be assigned a power spectrum which extends considerably beyond the passband of the unknown transfer function and is relatively flat within the passband. Thus in an ensemble sense the random process can apply frequency components to trace out the transfer function.

The power spectrum of the output process is a tracing of the magnitude squared of the unknown transfer function. This would be sufficient to determine a real, even impulse response, which has a real (and even) Fourier transform. A more arbitrary real impulse response with both odd and even components will correspond to a complex valued transfer function whose magnitude and phase must be traced out. The appropriate tracing is the cross power spectral density between input and output. Since this function is the product of the input power spectrum and the unknown transfer function, the use of a random signal with a whitish power spectrum will produce a cross-spectrum which essentially replicates the transfer function in magnitude and phase.

The corresponding relationship in the time domain is that the cross-correlation function between input and output of the impulse response is equal to the input autocorrelation function convolved with the impulse response. This provides the basis in an ensemble sense for determining the impulse response using a random signal. If the autocorrelation function of the random signal is narrow relative to the impulse response "duration" the cross-correlation function is a slightly smeared tracing of the impulse response.

In practice, what would be available for impulse response determination is a finite time segment of a particular sample function of the random process. Operationally, an "ergodic" assumption would be made, and the time cross-correlation function between the available segments of the input and output sample functions would be treated as the impulse response estimate.

D. Relationship of Approaches

We can now note a striking similarity with the use of the deterministic signal. In either case the received signal is cross-correlated with a replica of the transmitted signal. The resulting waveform is then the desired impulse response smeared by the auto-correlation function of the transmitted signal. So what then is the difference between using deterministic and random signals? In our view, there really isn't any, and there are several ways of looking at it.

Suppose first that a given segment of random process sample function, once generated, is utilized over and over again every time the radar attempts an impulse response determination. In this case the fact that the radar signal was originally generated by a random process becomes irrelevant.

It is now a deterministic signal. In practice its time duration would be long relative to its correlation time, and the operation of correlating it against a replica of itself would "pulse compress" it to a narrow spike whose width was the correlation time, riding above a sidelobe structure. The whole operation would closely parallel how things would go with a signal that had been obtained by deterministic means. A binary phase-coded pulse compression signal for example is really just a stylized version of a sample function of a continuous random process in which the process can change states abruptly after every "correlation interval" rather than continuously. The fact that such signals are often referred to as "pseudo-random" or "noise-like" reinforces the comparison.

The other possibility is that a segment of a new sample function is utilized each time the radar makes an impulse response determination. There would now be some variability due to the signal itself, but not a variability that seems significant. Stated differently, "most" sample function segments are equivalent if their time durations are long relative to the correlation time. That is, most sample functions, when correlated against themselves, will produce an essentially identical correlation function. There will be minute differences in the exact shape of the central spike and the details of the sidelobes, but in most cases the resulting impulse response estimate will be essentially unaffected.

If this line of heuristic reasoning seems at all convincing, one must ask why the notion has persisted that the use of a random signal has an advantage in impulse response determination. The answer is basically as follows. In the random signal case the operation was explicitly recognized as a cross-correlation operation. This led to the idea that any interference which was "uncorrelated" with the random signal would be suppressed when the signal and interference were cross-correlated. The textbook by Y. W. Lee[*] popularized this notion as far back as 1960. There are several problems with his discussion of this topic. Foremost is the fact that he apparently did not recognize that the "conventional" or deterministic signal method was really just as much of a cross-correlation method as the "novel" or random signal method. Secondly, he did not really deal with the fact that an uncorrelated disturbance will not completely vanish when subjected to a finite-time cross-correlation. The degree to which the interference would be suppressed was not examined and a comparison with the "old" method was not made.

---

*Y. W. Lee, Statistical Theory of Communication (John Wiley and Sons, New York, 1960).

The purpose of this introduction has been to engender an expectation that if the two methods were compared on the basis of equal utilization of resources such as energy and bandwidth, they would have essentially the same performance. A comparison has been carried out analytically, described in the following sections, which bears out this expectation.

II. Analysis

A. Framework

1. Performance Measure

Before proceeding with an analysis of each method and a comparison between them, we must establish a performance measure which will be the basis for comparison. Using either method, the impulse response estimate at any point on the true impulse response will be a random variable. In the deterministic signal method this is due to the interfering noise process, and in the random signal method it is due both to the interference and to the randomness of the signal itself. Thus the most general description of the estimators would be their multi-dimensional probability density functions at n points along the impulse response. If the ultimate goal is to describe how some discriminant which operates on the estimated impulse response performs, the number and location of the points at which the nth order density functions are evaluated would depend on the particular discriminant. For a length measurement, a few points near the front and back of each vehicle class might suffice if the signal/interference ratio were above a certain value. For a pattern recognition approach, points spread across the full extent of each impulse response might be necessary.

We do not want to consider specific discriminants in this study. To avoid doing so, we choose as a performance measure the normalized variance of the impulse response estimates at any point along the response. The notion is that any discriminant will benefit if this measure is small. We are saying that we want "most" of the random estimates produced by a given estimator to lie in a "narrow" (in a percentage error sense) band about the true impulse response pattern. We don't particularly care whether an estimated pattern looks as shown in Figure 1a, wherein the estimation errors quickly decorrelate across the pattern, or as shown in Figure 1b, wherein the errors are highly correlated across the pattern. Thus our performance measure is a first order statistic.

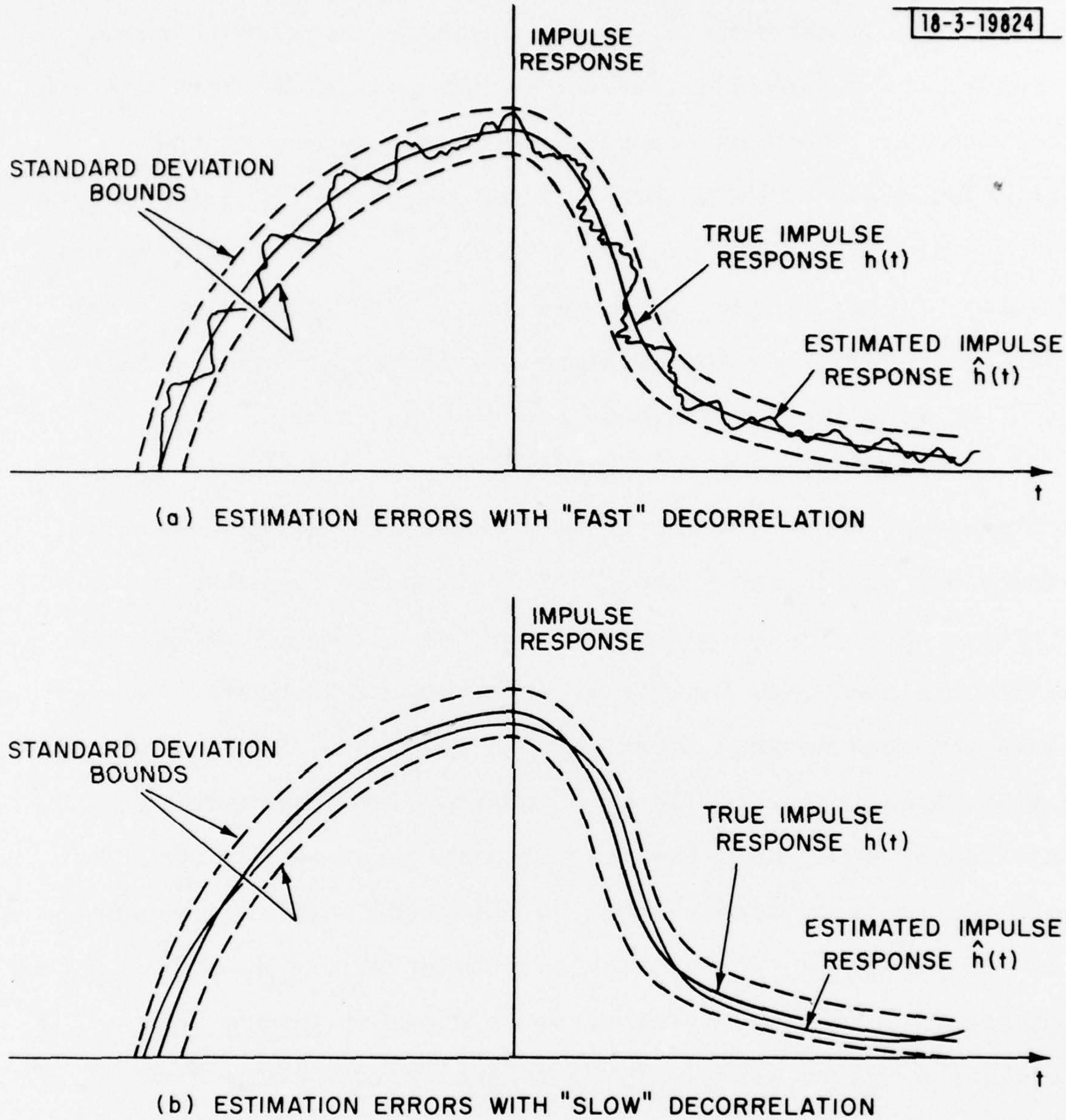


Fig. 1. Extremes of estimation error time behavior.

2. Model Considerations

In what follows, expressions are simplified if we calculate the normalized variance at certain "typical" points on the pattern, although the calculation can be made at any desired point. Simplification is also gained if the impulse response is assumed to have even symmetry. Again the restriction is made just for convenience. The analysis has been carried out for continuous impulse responses described by any one of a number of unimodal shapes. These include Gaussian, sin x/x, cusped exponential, and rectangular. The same shapes were used to describe the deterministic measurement signal and the auto-correlation function of the random measurement signal. The functional form of the results does not depend on the shape chosen in the operating regime of interest (see below). Only form factors such as $\pi, \sqrt{2}$, etc. change. The specific results in this report will correspond to the Gaussian shape assumption, and the typical point at which they are evaluated is the peak of the impulse response so shaped.

In addition to the basic results for an impulse response described by a smooth curve, some results are presented for an impulse response described by either one or a pair of impulses. The single impulse case corresponds to a point target model, and the results provide some insight in the continuous case. The impulse pair model, for which results are similar, corresponds to a situation in which resolution can be achieved between target features in an operational sense, but the bandwidth of the received signal is still determined by the transmitted signal bandwidth, not the target bandwidth. Such a model applies to a vehicle which generates radar returns primarily from a tip and a base such that the "rise time" of the signal is unaffected by the target at any signal bandwidth being considered. The typical point selected in the point target case is the location of the single impulse. In the impulse pair model two typical points are selected, one at an impulse location and one midway between impulses. Figure 2 illustrates the three cases.

3. Operating Regime

The results which follow below correspond to a certain operating regime for some of the basic time and bandwidth parameters involved. This regime is defined by the requirements that:

(i) the resolution of the estimate be somewhat finer than the time scale of the impulse response

(ii) the estimate be generated over a significant portion of the impulse response extent.

In our basic case of a continuous impulse response the first requirement amounts to saying that the signal bandwidth

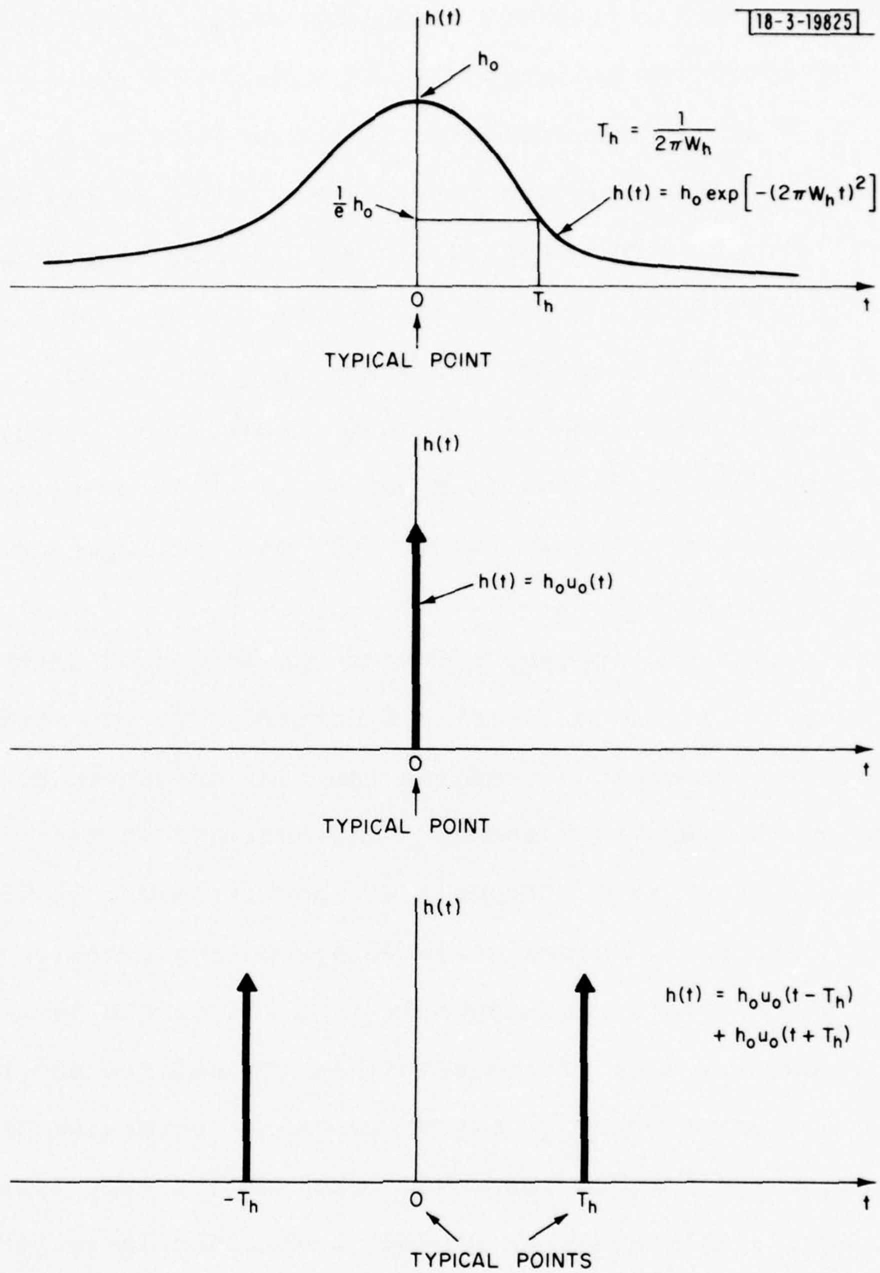


Fig. 2. Basic impulse response forms.

must be somewhat larger than the bandwidth of the system being measured. If we denote suitably defined signal and system bandwidths by W and W_h respectively (subscript "h" for impulse response h(t)), then we require that $W >> W_h$, where a reasonable operational definition of $>> W_h$ is $>10\ W_h$. Rather than describing the system by a bandwidth W_h, we can describe it by a suitably defined impulse response extent T_h, where $W_h T_h \simeq 1$. The requirement then becomes $WT_h >> 1$. In our impulse pair model, the system bandwidth W_h is infinite but T_h is still meaningful and the requirement to cleanly resolve the pair of impulses is again expressible as $WT_h >> 1$.

The second requirement concerns the amount of data that must be processed and is most easily described from the viewpoint of the random signal method. Imagine the time origin to be centered under the impulse response. The estimate of h(o) is then the cross-correlation between the transmitted and received waveforms at zero shift (propagation delays having been removed). To generate the estimate a distance T_h from the origin requires shifting the data by T_h. If the signal is transmitted and the data sample collected over $\pm T$, estimates can be generated only out to $\pm T$ along the impulse response. Thus at the very least, $T = T_h$. Further, unless the time window is somewhat longer than this the data falloff will be excessive and the window function will unduly affect the estimate at shifts approaching T_h. It is clearly desirable to require that $T >> T_h$, or $W_h T >> 1$.

It then follows from the first requirement that $WT \gg 1$ will also characterize the operating regime. The estimation accuracy expressions that result from an analysis taken to these limits on the parameters will show that desirable parameter settings from an accuracy viewpoint are consistent with the regime described here.

B. Random Signal Method

The random signal method can be described up to a point without specific reference to the form of the impulse response. The transmitted signal is a finite time segment of a sample function $x(t)$ of a real zero mean Gaussian random process. The signal exists for $|t| \leq T$, and a different sample function is used each time the radar makes a transmission. The power spectrum of $x(t)$ is assumed to be

$$S_x(f) = N_x \exp[-(f^2/2W^2)] \tag{1}$$

with peak spectral height N_x. The bandwidth parameter W has been chosen as the "standard deviation" of the Gaussian shape of the spectrum. The corresponding autocorrelation function of $x(t)$ (Fourier transform of $S_x(f)$) is

$$E[x(t)x(t+\tau)] = R_{xx}(\tau) = \sqrt{2\pi}\, WN_x \exp[-(2\pi^2 W^2 \tau^2)] \tag{2}$$

where E[·] denotes expectation. For large W, x(t) approaches white noise of spectral height N_x and

$$R_{xx}(\tau) \rightarrow N_x u_o(\tau) \tag{3}$$

where $u_o(\cdot)$ is the unit impulse. The average power in the process x(t) is

$$R_{xx}(o) = \sqrt{2\pi}\, W N_x \tag{4}$$

so that the average energy in a segment of x(t) 2T seconds long is

$$E_x = 2\,\sqrt{2\pi}\, W T N_x \tag{5}$$

a relation that will be used later

The reflected signal y(t) is the segment of x(t) convolved with the unknown impulse response h(t):

$$y(t) = \int_{-T}^{T} x(\xi)\, h(t-\xi)\, d\xi \tag{6}$$

The received signal is

$$z(t) = y(t) + n(t) \tag{7}$$

where n(t) is the interfering noise process, assumed to have the same statistics as x(t), to have spectral height N_n rather than N_x, and to be statistically independent of x(t) so that

$$E[x^k(t)\ n^j(t+\tau)] = E[x^k(t)]\ E[n^j(t+\tau)],\ \text{any } k,j,\tau \tag{8}$$

The estimator of h(t), found by cross-correlating z(t) with the transmitted segment of x(t) is

$$\hat{h}(\tau) = \frac{1}{2TN_x} \int_{-T}^{T} x(t)\ z(t+\tau)\ dt \tag{9}$$

where propagation delay to and from the target has been taken as zero. To calculate the mean and standard deviation of $\hat{h}(\tau)$ we must make some assumptions about $h(\tau)$.

1. Continuous Impulse Response

Assume that $h(\tau)$ is a smooth curve qualitatively described by an effective time duration T_h and corresponding bandwidth W_h, where

$$W_h T_h \simeq 1 \tag{10}$$

Then using Equations (6) through (9) one can show that

$$E[\hat{h}(\tau)] = \frac{1}{2TN_x} \int_{\xi=-T}^{T} \int_{\nu=(-T-\xi)}^{T-\xi} R_{xx}(\nu)h(\nu+\tau)\, d\nu\, d\xi \quad (11)$$

If we then assume that R_{xx} is narrow with respect to h, i.e., that

$$WT_h = W/W_h >> 1 \quad (12)$$

then (3) can be used to obtain

$$E[\hat{h}(\tau)] \simeq h(\tau) \quad (13)$$

so that $\hat{h}$ is an unbiased estimator of h.

The variance of $\hat{h}$ is

$$\mathrm{Var}[\hat{h}(\tau)] = E[(\hat{h}(\tau) - \overline{\hat{h}(\tau)})^2] \quad (14)$$

One proceeds by plugging (9) and (11) into (14). Appendix A describes what comes next. It involves such things as recognizing that a product of two multiple integrals can be written as a higher order integral by introducing additional dummy variables, the use of (8), and the use of the "moment theorem for Gaussian variables," which allows one to express the expected value of a product of three or more variables as a sum of expectations of products of two variables at a time. The result is that the desired variance can be written as:

$$\text{Var}[\hat{h}(\tau)] = \text{Var}_x[\hat{h}(\tau)] + \text{Var}_n[\hat{h}(\tau)] \tag{15}$$

The Var_x term depends only on $R_{xx}(\tau)$ and $h(\tau)$. It represents the variability in the estimate due to the randomness of the transmitted signal, and persists even if there is no interference whatsoever. The Var_n term depends only on $R_{xx}(\tau)$ and $R_{nn}(\tau)$, the autocorrelations of the signal and the interference. It corresponds to the fact that when the interference is cross-correlated against the transmitted process <u>for a finite time</u> the result has an expected value of zero, but a <u>non-zero</u> variance. The two terms are described in more detail below.

a. <u>Var_x Term</u>

If we again set $WT_h >> 1$, assume for convenience that $h(\tau)$ is even, and evaluate the variance at the typical point $\tau=0$, we obtain

$$\text{Var}_x[\hat{h}(o)] = \frac{1}{2T^2} \int_{\xi=-T}^{T} \int_{\nu=-T-\xi}^{T-\xi} h^2(\nu)\, d\nu\, d\xi \tag{16}$$

Next assume that $h(\tau)$ is also Gaussian shaped and write it as

$$h(\tau) = h_o \exp[-(2\pi W_h \tau)^2] \tag{17}$$

With this convention, the power transfer function of the target (magnitude squared of Fourier transform of $h(\tau)$) is

$$|H(f)|^2 = H_o^2 \exp[-(f^2/2W_h^2)],$$
$$H_o^2 = \frac{h_o^2}{4\pi W_h^2} \tag{18}$$

A comparison with (1) shows that the bandwidth parameters W and W_h have been defined consistently. That is, the random processes $x(t)$ and $n(t)$ can be thought of as white noise passed through a filter with a Gaussian shaped passband $|H_{x,n}(f)|^2$ whose "variance parameter" is W^2, while the target is a filter with Gaussian shaped passband $|H(f)|^2$ whose "variance parameter" is W_h^2. The generic parameter T_h used above can be thought of as $\simeq 1/W_h$. Using (17) in (16) and setting $W_hT >> 1$ results in a normalized variance given by

$$\boxed{\frac{\mathrm{Var}_x[\hat{h}(o)]}{h^2(o)} = \frac{1}{2\sqrt{2\pi}} \; \frac{1}{W_hT}} \tag{19}$$

where the first factor would vary if the definition of bandwidth or the shape assumed for $h(\tau)$ changed, and the second factor expresses the fundamental dependence involved. Equation (19) has an interesting interpretation that will be deferred to Section B-2 below.

b. Var_n Term

This term results when the procedure described just before Equation (15) is carried out. It corresponds to the expectation

$$E\left\{\left[\frac{1}{2TN_x}\int_{-T}^{T} x(t)n(t+\tau)dt\right]^2\right\}$$

$$= E\left[\frac{1}{4T^2N_x^{\ 2}}\iint_{-T}^{T} x(t)x(\xi)n(t+\tau)n(\xi+\tau)\ dtd\xi\right] \tag{20}$$

If the expectation of the integrand is taken, the independence of x and n invoked, and the variable $\rho=\xi-t$ introduced, the result is

$$\frac{\mathrm{Var}_n[\hat{h}(\tau)]}{h^2(o)} = \frac{1}{h_o^{\ 2}\cdot 4T^2N_x^{\ 2}}\int_{t=-T}^{T}\int_{\rho=-T-t}^{T-t} R_{xx}(\rho)R_{nn}(\rho)d\rho dt \tag{21}$$

If (2) is then applied and the result evaluated for $WT \gg 1$ we arrive at the normalized variance

$$\frac{\mathrm{Var}_n[\hat{h}(o)]}{h^2(o)} = \frac{\sqrt{\pi}}{2} \; \frac{W}{T} \; \frac{N_n}{N_x} \tag{22}$$

where we have let $h_o=1$. This entails no loss of generality if we think of N_x as representing the transmitted power level modified by radar range equation factors, one of which is target cross-section level, i.e., h_o. Finally, if we apply (5), the result can be written

$$\boxed{\frac{\mathrm{Var}_n[\hat{h}(o)]}{h^2(o)} = \frac{\sqrt{2}\,\pi\,W^2}{E_x/N_n}} \tag{23}$$

These results can be interpreted readily for either form (22) or (23). To understand (22) one can go back to (20) and think of the integral as a sum of independent random variables, 4TW in number, spaced by $\frac{1}{2W}$ in time, each being the product of a sample of x, a sample of n, and the spacing factor 1/2W. Squaring the sum, taking the expectation, and using (8) results in a new summation in which each term contains the expected value of the square of a sample of x times the square of a sample of n. Applying (4) then results in

$$\mathrm{Var}_n' = \frac{\pi}{2}\,\frac{W}{T}\,\frac{N_n}{N_x} \tag{24}$$

which agrees with (22) to within a factor of $\sqrt{\pi}$, the difference corresponding to the fact that the integral form is evaluated for the actual Gaussian shape of the correlation functions while the summation form uses a rectangle approximation. If we then write the factor W/T as W^2/TW we see that Var_n' is directly proportional to W^2, inversely proportional to TW, and inversely proportional to N_x/N_n. These factors are respectively the bandwidth dependence of the variance of each term in the original sum, the dependence of the number of independent samples on time and bandwidth, and the power ratio of the "signal" and interference samples. These identifications constitute our interpretation of (22).

Regarding form (23), the inverse dependence on a "signal energy to noise power density" ratio is not surprising since we are dealing with the normalized variance of a signature amplitude. The increase of variance as W^2 results from the fact that we are operating with $W \gg W_h$ to achieve resolution. One factor of W corresponds to a loss of energy outside the nominal target bandwidth W_h. The other factor of W corresponds to a mismatch loss at the receiver because the receiver must operate with bandwidth W to achieve the resolution while the power bandwidth of the received signal is essentially W_h. This explanation is repeated more thoroughly in Section II C-1 below.

2. Impulsive Impulse Response

In this section we no longer think of h(t) as being a smooth curve but rather assume that it is composed of either one or two impulses. As discussed in Section II A-2, the single impulse represents an idealized point target, and provides a comparison with the continuous case. The two impulse model is quite similar, and represents a target that returns from a "tip" and a "base" with essentially no broadening of either return at any operating bandwidth being considered. The two models are shown in Figure 3a. For simplicity the two impulses are assumed to have equal areas and to sit symmetrically about the time origin.

Equations (1) through (9) and (11) still apply. Studying (11) indicates that in the point target case the expected value of the estimate follows the shape $R_{xx}(\tau)$, which is a "narrow" continuous pulse with width inversely proportional to W rather than a spike. In the two impulse case, $E[\hat{h}(\tau)]$ looks essentially like a pair of offset replicas of $R_{xx}(\tau)$. If 1/W is small compared to the impulse spacing, i.e., if $WT_h \gg 1$, the two peaks are well resolved and $E[\hat{h}(\tau)]$ looks like a pair of "narrow" continuous peaks. Figure 3b shows these curves. If we proceed from (11) using (2) and assume that $WT_h >> 1$ and $WT >> 1$, we obtain

$$\begin{aligned} E[\hat{h}(\tau)] &= \sqrt{2\pi}\, Wh_o, \qquad \tau \text{ at an impulse location} \\ E[\hat{h}(\tau)] &= 0, \quad \tau \text{ "far" from an impulse location} \end{aligned} \tag{25}$$

where h_o is the area of each impulse. The first of these values will be used to normalize the variance terms to follow.

a. Var_x Term

Equation (15) still applies, stating that the variance of the estimate is the sum of a Var_x and a Var_n term. However, the Var_x term is no longer given by (16). The problem must be reworked starting with (11) and (14) as in Appendix A. If this is done utilizing (2) and the model of Figure 3a, assuming that WT_h and WT are $>> 1$, and letting h_o be unity, we obtain in the point target case

$$\boxed{\begin{aligned} \frac{Var_x[\hat{h}(\tau)]}{(E[\hat{h}(o)])^2} &= \frac{1}{2\sqrt{\pi}\, WT} \qquad \text{at impulse location} \\ &= \frac{1}{4\sqrt{\pi} WT} \qquad \text{"far" from impulse} \end{aligned}} \tag{26}$$

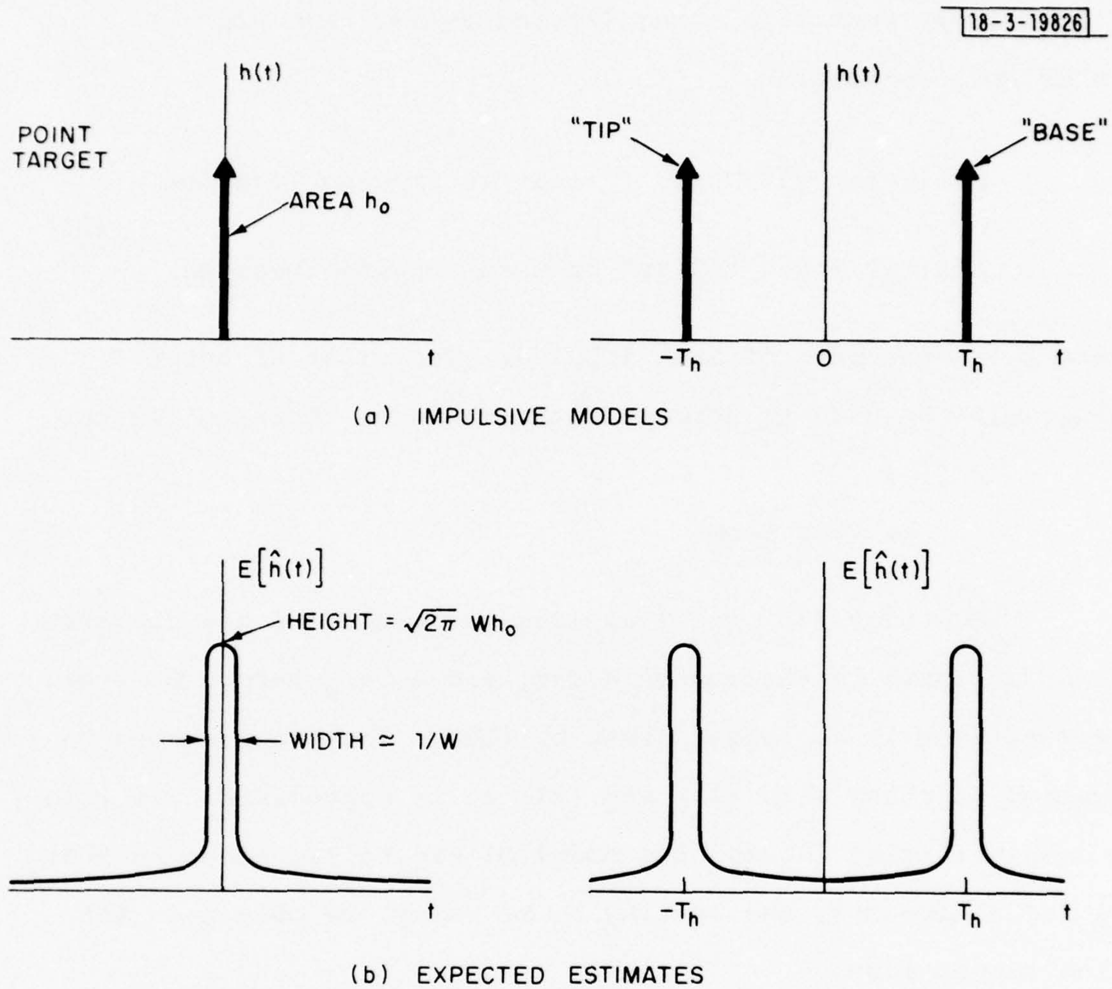


Fig. 3. True and expected impulse response shapes for impulsive case.

In the two impulse case we obtain

$$\frac{\mathrm{Var}_x[\hat{h}(\tau)]}{(E[\hat{h}(T_1)])^2} = \frac{3}{4\sqrt{\pi}\,WT} \qquad \text{at impulse locations}$$

$$= \frac{1}{\sqrt{\pi}WT} \qquad \text{between impulses} \tag{27}$$

The slight differences in the numerical coefficients can be explained but we will not digress to do so here. The interesting thing to observe is the inverse dependence on time-bandwidth product. The product WT is proportional to the number of independent samples of the transmitted process $x(t)$ in the time interval $(-T,T)$. Since an impulsive impulse response does not limit the bandwidth of the reflected signal $y(t)$, the product $x(t)y(t)$ is essentially proportional to $x^2(t)$ and still has on the order of WT independent samples. Equations (7) and (9) indicate that Var_x is the estimate variance with the noise $n(t)$ set to zero, which is proportional to the variance of essentially an average of a set of samples of $x^2(t)$. In normalized form this varies inversely with the number of independent samples involved, i.e., WT. The result can be verified by approximating the integral in (9) by a sum of independent samples, as we did in Section II B-1-b above.

Now compare these results with those for the continuous impulse response, Equation (19). We see that basically where W appears in the impulsive result, W_h appears in the continuous result. What is happening is this. In the continuous case, under the assumption that $W \gg W_h$, the bandwidth of the reflected signal y(t) is limited to W_h by the frequency response of the target. Thus the number of independent samples of y(t) available is proportional to $W_h T$, not WT, and this change carries through to the result (19). Again, this inverse $W_h T$ dependence can be verified by a discrete summation approach, although it is slightly more complicated than verifying the WT dependence in the impulsive case.

b. Var_n Term

Sine the Var_n term comes about from cross-correlating n(t) with a replica of x(t), it does not depend on h(t), and hence in un-normalized form is unchanged from the continuous case. However, the normalized form, which is what counts, will change in accordance with (25), and becomes, using (22) and (25),

$$\frac{\mathrm{Var}_n[\hat{h}(\tau)]}{(E[\hat{h}(\tau_o)])^2} = \frac{1}{4\sqrt{\pi}} \frac{1}{WT} \frac{N_n}{N_x} \tag{28}$$

which, using (5), can be written

$$\frac{\mathrm{Var}_n[\hat{h}(\tau)]}{(E[\hat{h}(\tau_o)])^2} = \frac{\sqrt{2}}{2} \frac{1}{E_x/N_n} \tag{29}$$

where τ_o represents the location of an impulse in $h(\tau)$.

Comparing (29) with (23) we see that the growth of variance with bandwidth no longer occurs in the impulsive case. This is because the target frequency response no longer limits the bandwidth of the reflected signal and causes energy loss and mismatch at the receiver. Again, this point will be made more explicitly in Section II C-2 below.

C. Deterministic Signal Method

The basis for the deterministic signal method is shown in Figure 4. The transmitted signal $s(t)$ is to be thought of as a real "narrow" pulse with energy E given by

$$E = \int_{\infty}^{\infty} s^2(t)\,dt \tag{30}$$

To remain consistent with Section B, the signal will be assigned the Gaussian shape

$$s(t) = s_o \exp[-(2\pi Wt)^2] \tag{31}$$

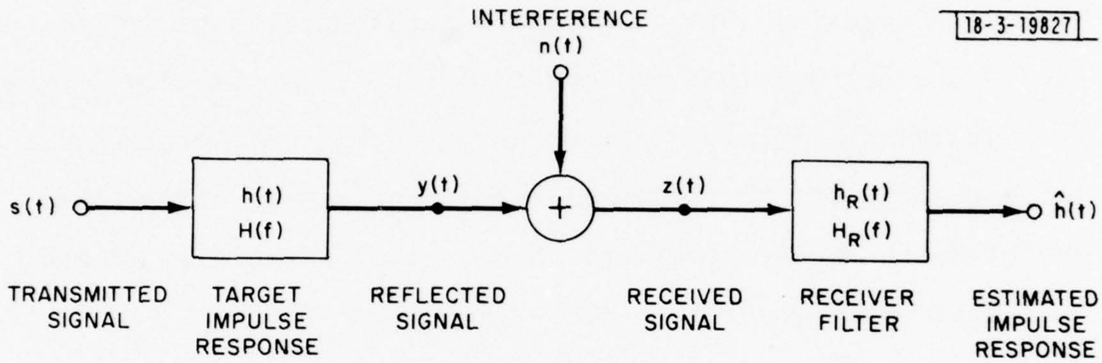


Fig. 4. Deterministic signal method.

with Fourier transform

$$S(f) = S_o \exp[-(f/2W)^2] \qquad (32)$$

where one can show that

$$s_o^2 = 4\pi W^2 S_o^2 = \sqrt{8\pi}\ EW \qquad (33)$$

The fact that the signal extends to infinity and that the convolutions to follow are carried out with infinite limits is not significant. The same analysis has been carried out with signals and filters having finite time extent, rectangular shapes for example, with no change in the basic dependences, merely form factor variations. Truncating the Gaussian shapes used here would change the results negligibly while introducing much nuisance value into the analysis.

The reflected signal y(t) is s(t) convolved with h(t), or

$$y(t) = \int_{\infty}^{\infty} s(\xi)h(t-\xi)d\xi \qquad (34)$$

which will henceforth be denoted by $s(t) \otimes h(t)$. The received signal is

$$z(t) = y(t) + n(t) \qquad (35)$$

which is then passed through a receiver filter whose impulse response is $h_R(t)$ resulting in a waveform which acts as the estimate of $h(t)$. Thus,

$$\hat{h}(t) = z(t) \otimes h_R(t) = s(t) \otimes h(t) \otimes h_R(t) + \underbrace{n(t) \otimes h_R(t)}_{m(t)} \tag{36}$$

The interference $n(t)$ is as assumed in Section B, with Gaussian shaped spectrum $S_n(f)$ and autocorrelation $R_{nn}(\tau)$. Equations (1) through (3) apply with N_n replacing N_x. The receiver filter is taken to be a filter matched to $s(t)$, and therefore has an impulse response proportional to $s(-t)$. Thus we can write

$$\begin{aligned} h_R(t) &= h_{Ro} \exp[-(2\pi W t)^2] \\ H_R(f) &= H_{Ro} \exp[-(f/2W)^2] \\ h_{Ro}^{\ 2} &= 4\pi W^2 H_{Ro}^{\ 2} \end{aligned} \tag{37}$$

We can now calculate the mean and variance of $\hat{h}(t)$ for the continuous and impulsive impulse response models.

1. Continuous Impulse Response

Assume again that the target impulse response is

Gaussian shaped with bandwidth parameter W_h, as described by Equations (17) and (18). From (36) we can write

$$E[\hat{h}(t)] = s(t) \otimes h(t) \otimes h_R(t) = s(t) \otimes h_R(t) \otimes h(t) \qquad (38)$$

From (31), (32), and (37) we can write

$$s(t) \otimes h_R(t) = \sqrt{2\pi}\, W S_o H_{Ro} \exp[-(\sqrt{2}\pi W t)^2] \qquad (39)$$

For large W,

$$s(t) \otimes h_R(t) \rightarrow S_o H_{Ro} u_o(t) \qquad (40)$$

Thus, if $W \gg W_h$, we can write using (38) and (40),

$$E[\hat{h}(t)] = S_o H_{Ro} h(t) \qquad (41)$$

Using (36) we can write

$$\mathrm{Var}[\hat{h}(t)] = E[m^2(t)] = R_{mm}(o) = \int_{\infty}^{\infty} S_m(f)\,df \qquad (42)$$

where m(t) is the noise at the output of the receiver filter, $R_{mm}(\tau)$ is its autocorrelation and $S_m(f)$ is its power spectrum. Therefore

$$\mathrm{Var}[\hat{h}(t)] = \int_{\infty}^{\infty} S_n(f)\,|H_R(f)|^2\,df \tag{43}$$

This evaluates to

$$\mathrm{Var}[\hat{h}(t)] = \sqrt{\pi}\, W N_n H_{Ro}^{\;2} \tag{44}$$

and using (33) and (41), letting $h_o=1$, and $W >> W_h$, we can write

$$\boxed{\frac{\mathrm{Var}[\hat{h}(o)]}{(E[\hat{h}(o)])^2} = \frac{\sqrt{2}\,\pi\, W^2}{E/N_n}} \tag{45}$$

To understand this result, consider the "narrowband" limit $W << W_h$. In this case the only effect of convolving $(s(t) \otimes h_R(t))$ with $h(t)$ is to multiply $(s(t) \otimes h_R(t))$ by the amplitude of the target frequency response at zero frequency, namely H_o. Thus from (38) and (39) we can write

$$E[\hat{h}(o)] = \sqrt{2\pi}\, W S_o H_{Ro} H_o \tag{46}$$

and using (44), (46), (33), and (18), and letting $h_o=1$, we obtain the normalized result

$$\frac{\mathrm{Var}[\hat{h}(o)]}{(E[\hat{h}(o)])^2} = \frac{2\sqrt{2}\,\pi W_h^{\;2}}{E/N_n} \tag{47}$$

Thus the ratio of the normalized variances in the wideband and narrowband limits is

$$\frac{Var_{Wb}}{Var_{nb}} = \frac{1}{2}\left(\frac{W}{W_h}\right)^2 \tag{48}$$

Two effects contribute to this result. In the narrowband case the receiver is matched to the reflected signal y(t). In the wideband case the bandwidth of the reflected signal has been limited by the target, while the receiver operates wideband to match the transmitted signal. Thus excess noise enters the receiver relative to the matched case, the effect being proportional to W/W_h. The other effect is that the target rejects most of the transmitted energy outside its nominal bandwidth W_h. Energy must be transmitted "out of band" in order for the expected pattern of the estimate to contain the desired detail or resolution, but most of this energy is lost from the standpoint of the variability of the estimate. This effect is also proportional to W/W_h.

We see then that "too much" bandwidth is a bad thing, using either the deterministic or the random signal. If you ask for more resolution or detail than you really need, a noisier estimate results.

2. Impulsive Impulse Response

Assume again that the target impulse response is not a continuous curve, but contains impulses. For simplicity, the single impulse or point target model will be treated. Thus h(t) is taken to be

$$h(t) = h_o u_o(t) \tag{49}$$

Then from (38) we can write

$$E[\hat{h}(t)] = h_o s(t) \otimes h_R(t) \tag{50}$$

which says that the expected shape of the estimate, ideally an impulse, is the shape of the pulse at the output of the receiver filter. Applying (39) and (33) we get

$$(E[\hat{h}(o)])^2 = 2\pi\ W^2 S_o{}^2 H_{Ro}{}^2 h_o{}^2 = \sqrt{2\pi}\ EWH_{Ro}{}^2 h_o{}^2 \tag{51}$$

The un-normalized variance is still given by (44), and the resulting normalized expression is

$$\frac{\mathrm{Var}[\hat{h}(o)]}{(E[\hat{h}(o)])^2} = \frac{\sqrt{2}}{2}\ \frac{1}{E/N_n} \tag{52}$$

III. Conclusions

In Section I we addressed heuristically the question of whether the use of random rather than deterministic signals had any advantage for impulse response estimation. In Section II we defined an objective basis for making the comparison, carried the comparison out, and developed some insight into the results. In this section we will summarize what has been learned and briefly comment on it.

The basic performance expressions derived are brought together in Table 1 below.

TABLE 1

NORMALIZED VARIANCE OF IMPULSE RESPONSE ESTIMATE

	CONTINUOUS IMPULSE RESPONSE		IMPULSIVE IMPULSE RESPONSE	
	VARIANCE DUE TO SIGNAL	VARIANCE DUE TO INTERFERENCE	VARIANCE DUE TO SIGNAL	VARIANCE DUE TO INTERFERENCE
RANDOM SIGNAL	$\frac{1}{2\sqrt{2\pi}} \quad \frac{1}{W_h T}$	$\sqrt{2\pi} \quad \frac{W^2}{E/N_n}$	$\frac{1}{2\sqrt{\pi}} \quad \frac{1}{WT}$	$\frac{\sqrt{2}}{2} \quad \frac{1}{E/N_n}$
DETERMINISTIC SIGNAL	NONE	$\sqrt{2\pi} \quad \frac{W^2}{E/N_n}$	NONE	$\frac{\sqrt{2}}{2} \quad \frac{1}{E/N_n}$

These results apply in the limits on time and bandwidth previously discussed, and the numerical coefficients correspond to the Gaussian shape assumptions and definition of bandwidth made. The important point is that bandwidth has been defined consistently in the sense that a given value of W implies exactly the same resolution or detail in the expected estimate in either the deterministic or the random case.

The table shows clearly what the answer is regarding the central question addressed in this report, namely the relative ability of the two methods to suppress uncorrelated interference. Their performance is identical for either type of impulse response model when they employ equal resources such as bandwidth, average energy consumption, etc.

As described, the random signal method experiences an additional variance component, although this can be reduced to a desired level by adjusting the parameters. Alternately one can select a particular realization of the random process and use it repeatedly as a radar signal just like any other deterministic signal, e.g., LFM, binary phase coded, etc.

Finally, we have pointed out that with either type of signal the use of "excessive" bandwidth carries with it the penalty of a more variable estimate. This should be kept in mind in discrimination applications where achieving fine resolution on the shortest targets in question may in fact be unnecessary or even undesirable.

APPENDIX

The purpose of the appendix is to describe in somewhat more detail than was appropriate in Section II how some of the variance expressions are arrived at. In the random signal case we start with equation (14) giving the definition of the estimator variance as

$$\mathrm{Var}[\hat{h}(\tau)] = E[(\hat{h}(\tau) - \overline{\hat{h}(\tau)})^2] \tag{A1}$$

Using the definition of $\hat{h}(\tau)$ from Equation (9),

$$\hat{h}(\tau) = \frac{1}{2TN_x} \int_{-T}^{T} x(t) z(t+\tau)\, dt \tag{A2}$$

the expression given in (11) for the expected value of $\hat{h}(\tau)$,

$$E[\hat{h}(\tau)] = \frac{1}{2TN_x} \int_{\xi=-T}^{T} \int_{\nu=-T-\xi}^{T-\xi} R_{xx}(\nu) h(\nu+\tau) d\nu\, d\xi \tag{A3}$$

and the signal relationships from (6) and (7),

$$z(t) = y(t) + n(t) \tag{A4}$$

$$\text{and} \quad y(t) = \int_{-T}^{T} x(\xi)\, h(t-\xi)\, d\xi \tag{A5}$$

we can write

$$\hat{h}(\tau) - \overline{\hat{h}(\tau)} = \frac{1}{2TN_x} \int_{\xi=-T}^{T} \int_{\nu=-T-\xi}^{T-\xi} [x(\xi)x(\xi+\nu) - R_{xx}(\nu)]h(\nu+\tau)d\nu d\xi$$

$$+ \frac{1}{2TN_x} \int_{-T}^{T} x(\rho)n(\rho+\tau)d\rho \tag{A6}$$

Think of the first (double) integral as a term "A" and the second integral as a term "B". Then the square of the left side of (A6) is equal to $(A+B)^2$ which equals $A^2 + 2AB + B^2$. The "A^2" term is the square of the double integral and can therefore be written

$$A^2 = \frac{1}{4T^2N_x^{\ 2}} \int_{\xi=-T}^{T} \int_{\nu=-T-\xi}^{T-\xi} \int_{t=-T}^{T} \int_{\rho=-T-t}^{T-t} [x(\xi)x(\xi+\nu)-R_{xx}(\nu)]$$

$$[x(t)x(t+\rho)-R_{xx}(\rho)]\ h(\nu+\tau)h(\rho+\tau)\ d\xi d\nu dt d\rho \tag{A7}$$

The "B^2" term is the square of the single integral and can therefore be written

$$B^2 = \frac{1}{4T^2N_x^{\ 2}} \iint_{-T}^{T} x(\rho)x(t)n(\rho+\tau)n(t+\tau)d\rho dt \tag{A8}$$

The cross term "2AB" is the product of the single and double integrals, which can be written

$$2AB = \frac{1}{2T^2 N_x^{\ 2}} \int_{\xi=-T}^{T} \int_{\nu=-T-\xi}^{T-\xi} \int_{\rho=-T}^{T} [x(\xi)x(\xi+\nu)-R_{xx}(\nu)] \tag{A9}$$

$$x(\rho)n(\rho+\tau)h(\nu+\tau)\,d\rho\, d\nu\, d\xi$$

The expressions (A7) through (A9) comprise the quantity $[\hat{h}(\tau) - E(\hat{h}(\tau)]^2$. The variance of $\hat{h}(\tau)$ is then the sum of their expected values. In each case, the expectation is moved inside the integral. The first thing we note is that because of the independence of x and n, the entire integrand in (A9) is proportional to $E[n(\rho+\tau)]$ which is zero (see Equation (8), Section II B). Thus (A9) contributes nothing, and we have only (A7) and (A8) to consider.

The expected value of (A8) is the Var_n term. If we make the change of variables $\xi= \rho-t$, we can rewrite (A8) as

$$Var_n[\hat{h}(\tau)] = \frac{1}{4T^2 N_x^{\ 2}} \int_{t=-T}^{T} \int_{\xi=-T-t}^{T-t} E[x(t)x(t+\xi)n(t+\tau)n(t+\tau+\xi)] \tag{A10}$$

$$d\xi dt$$

Again using (8) because x and n are independent, we arrive at

$$\mathrm{Var}_n[\hat{h}(\tau)] = \frac{1}{4T^2 N_x{}^2} \int_{t=-T}^{T} \int_{\xi=-T-t}^{T-t} R_{xx}(\xi)\, R_{nn}(\xi)\, d\xi dt \qquad \text{(A11)}$$

This is the basic result for Var_n. If we substitute the particular Gaussian shapes assumed in Section II for R_{xx} and R_{nn}, the integration can be carried out in closed form, resulting in an expression involving the error function "erf." When this expression is evaluated for large WT, expression (22) for Var_n results.

The expected value of (A7) is the Var_x term. The procedure is as follows. The expectation is moved inside the integral and the integrand is multiplied out. Three of the four terms that result are of the form $R_{xx}(\nu)R_{xx}(\rho)h(\nu+\tau)h(\rho+\tau)$. The fourth term is of the form $E\left[x(\xi)x(\xi+\nu)x(t)x(t+\rho)\right] h(\nu+\tau)h(\rho+\tau)$. The expectation can be calculated by applying the "moment theorem" for Gaussian variables", which can be stated in the form

$$E[\alpha\beta\gamma\delta] = E[\alpha\beta]\ E[\gamma\delta] + E[\alpha\gamma]\ E[\beta\delta] + E[\alpha\delta]\ E[\beta\gamma] \qquad \text{(A12)}$$

where α, β, γ, and δ are real zero mean Gaussian random variables. If (A12) is applied, one of the three terms that results again contains the factor $R_{xx}(\nu)R_{xx}(\rho)$, and just cancels the previous terms containing that factor. Thus all that remains are the other two terms that result from applying (A12), and we arrive at

$$\mathrm{Var}_x[\hat{h}(\tau)] = \frac{1}{4T^2N_x^2} \int_{\xi=-T}^{T} \int_{\nu=-T-\xi}^{T-\xi} \int_{t=-T}^{T} \int_{\rho=-T-t}^{T-t}$$

$$[R_{xx}(\xi-t)R_{xx}(\xi-t+\nu-\rho) + R_{xx}(t+\rho-\xi)R_{xx}(\xi+\nu-t)] \tag{A13}$$

$$h(\nu+\tau)h(\rho+\tau)\,d\rho\, dt\, d\nu\, d\xi.$$

This is the basic result for Var_x, and it is not very informative as it stands. It remains to turn it into a more useful form in both the continuous and impulsive impulse response cases.

In the continuous case the assumption that R_{xx} is narrow relative to h,($WT_h >> 1$), allows us to make the substitution $R_{xx}(\cdot) \rightarrow N_x u_o(\cdot)$ in (A13). If we do this and then integrate first over ρ and then over t, the result is

$$\mathrm{Var}_x[\hat{h}(\tau)] = \frac{1}{4T^2} \int_{\xi=-T}^{T} \int_{\nu=-T-\xi}^{T-\xi} h(\nu+\tau)[h(\nu+\tau) + h(\tau-\nu)]\, d\nu\, d\xi \tag{A14}$$

and if we assume for convenience that h is even and evaluate at the typical point $\tau=0$, the result becomes

$$\mathrm{Var}_x[\hat{h}(o)] = \frac{1}{2T^2} \int_{\xi=-T}^{T} \int_{\nu=-T-\xi}^{T-\xi} h^2(\nu)d\nu \qquad \text{(A15)}$$

which is Equation (16) of Section II.

In the impulsive case, we have by assumption that h is impulsive relative to R_{xx}, whereas in the continuous case we treated R_{xx} as impulsive relative to h in the limit of interest. If we return to (A13), integrate first on ν, then on ρ, and assume here for simplicity the point target model $h(\tau) = h_o u_o(\tau)$, we obtain

$$\mathrm{Var}_x[\hat{h}(\tau)] = \frac{h_o{}^2}{4T^2 N_x{}^2} \iint_{-T}^{T} R_{xx}(t-\xi-\tau)R_{xx}(t-\xi+\tau)\ d\xi dt \qquad \text{(A16)}$$

If we evaluate at the typical point $\tau=0$, and change variables we arrive at

$$\mathrm{Var}_x[\hat{h}(o)] = \frac{h_o{}^2}{4T^2 N_x{}^2} \int_{\xi=-T}^{T} \int_{\eta=-T-\xi}^{T-\xi} R_{xx}{}^2(\eta)\ d\eta d\xi \qquad \text{(A17)}$$

This can be evaluated just like (A11), and Equation (26) results.

Acknowledgment

I want to thank Linda Theobald for a truly exemplary job in preparing this report. The sum of her speed, accuracy, and pleasantness probably exceeded known upper bounds previously reported in the literature.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

19

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 18 ESD-TR-77-344	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): 6 A Comparison of the Use of Deterministic and Random Signals for Impulse Response Determination.		5. TYPE OF REPORT & PERIOD COVERED: 9 Technical Note 6. PERFORMING ORG. REPORT NUMBER: Technical Note 1977-49
7. AUTHOR(s): 10 Dennis H. Pruslin 14 TN-1977-49		8. CONTRACT OR GRANT NUMBER(s): 15 F19628-78-C-0002
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Lincoln Laboratory, M.I.T. P.O. Box 73 Lexington, MA 02173		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: Project No. 8X363304D215 16
11. CONTROLLING OFFICE NAME AND ADDRESS: Ballistic Missile Defense Program Office Department of the Army 5001 Eisenhower Avenue Alexandria, VA 22333		12. REPORT DATE: 11 27 December 1977 13. NUMBER OF PAGES: 52
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): Electronic Systems Division Hanscom AFB Bedford, MA 01731 12 51p.		15. SECURITY CLASS. (of this report): Unclassified 15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

None

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

cross-correlation	random signals	wideband discrimination
impulse response	noise	length measurement
linear system	matched filter	deterministic signals

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A comparison is made between the use of deterministic and random signals for estimating the impulse response of a linear system. It is shown that the two methods are essentially identical, and that in particular, the random signal method has no advantage from the viewpoint of suppressing the effects of uncorrelated interference.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

207 650

LB